# 1 Electronic Supplementary Material

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## 3 Flickering as an early warning signal

4 Vasilis Dakos, Egbert H. van Nes, and Marten Scheffer

### 5 Online Resource 1

Figure A1 Stationary distributions based on a single simulation. For comparison we juxtaposed
trends in the indicators derived by slowing down (a). Only standard deviation increased before
the system started flickering both in the driver-mediated (b, b1) and noise-mediated scenario (c1).
Autocorrelation at-lag-1 (b2), and skewness (b3) showed no sign of change in the drivermediated scenario, but both spiked only during flickering and subsided after the disappearance of
the oligotrophic attractor. The system behaved similarly in the noise-mediated case (c2, c3),

12 except that skewness rose smoothly also before flickering.



14 Figure A2 Indicators from the total 200 Monte Carlo realizations for stationary distribution 15 experiments. Results are very similar to the ones derived for transient simulations (Fig. 3 main 16 text). In the case of the driver-mediated flickering (b), an increase in phosphorus input rate up to 17 the appearance of the alternative attractor caused a clear rise in standard deviation (b1), while no 18 apparent change was observed in autocorrelation at-lag-1 (b2) and in skewness (b3). Signs of 19 increasing autocorrelation and skewness were observed only shortly before the system started to 20 wander further into the alternative basin of attraction (at input rates between 0.4 and 0.45). After 21 this point the strongest rise in all indicators was documented (b1-3), which actually signaled that 22 the system was visiting the alternative state. In the case of the noise-mediated flickering (c), for 23 low noise magnitudes, standard deviation gradually increased (c1), autocorrelation staved almost 24 constant (c2), while skewness rose (c3). An increase in autocorrelation was observed only when 25 noise was becoming strong enough to push the system more towards the unstable equilibrium and even to cross into the alternative basin of attraction. As noise increased further only 26 27 autocorrelation decreased (c2) similar to autocorrelation in the driver-mediated flickering (b2). 28 Skewness remained at a high level (c3), and standard deviation kept increasing (c1) following the 29 increasing noise magnitude.



Figure A3 In the noise-induced scenario an increase in noise magnitude in a system outside the bistability region (a) could not be identified in the trends in the indicators characteristic to flickering. Only when the system is just after the bistability region (b) or well into the bistability region (c) the patterns would confirm the existence of the alternative attractor (panel b, c).



Figure A4 In the noise-induced scenario an increase in noise magnitude in a system outside the bistability region (a) could not map an alternative attractor in the reconstructed potential. The alternative attractors would start to be detected at the onset of bistability (b), but only when the system is well into the bistability region the patterns would unequivocally confirm the existence of the alternative attractor (c).



### 43 **Online Resource 2 Flickering in space**

We also explored flickering in space. It has been suggested that estimating spatial indicators
derived from spatial datasets may provide better signal for identifying an upcoming transition
(Guttal and Jayaprakash 2009; Scheffer et al. 2009; Carpenter and Brock 2010; Dakos et al. 2010,
2011), but no study on leading indicators so far has looked into the effects of flickering (or strong
noise) in a spatially extended system.

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50 To study flickering in a spatially explicit eutrophication scenario, we assumed that the dynamics 51 take place in discrete  $n \times n$  evenly spaced sites in the lake (Keitt et al. 2001; Van Nes and Scheffer 2005) coupled through a dispersion term D (Okubo 1980) that allows phosphorus to 52 53 diffuse between neighboring sites. Each site can individually switch to its alternative state and is 54 connected with its four neighbors. Spatial heterogeneity in the lake (for instance local hydrological differences) was introduced randomizing recycling rate  $r_{i,i}$  in each site. We also 55 56 assumed that noise was introduced independently across sites and we defined periodic boundaries 57 to prevent edge effects. Thus, the spatial version of the model reads:

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$$dP_{i,j} = [\alpha - sP_{i,j} + r_{i,j} \frac{P_{i,j}^8}{P_{i,j}^8 + 1} + D(P_{i+1,j} + P_{i-1,j} + P_{i,j+1} + P_{i,j-1} - 4P_{i,j})]dt + \sigma P_{i,j}dW_{i,j}$$
(eqA1).

We started simulations from the oligotrophic state and increased gradually the phosphorus input rate  $\alpha$  in 100 steps from 0.2 to 0.8. At each step we simulated for 200 time steps and we used the last values of phosphorus concentration in each site of the whole grid (50 × 50 sites) to calculate spatial as standard deviation (estimated as standard deviation, (Dakos et al. 2011)), spatial correlation of neighboring cells (Dakos et al. 2010), and spatial skewness (Guttal and Jayaprakash 2009). Also we used the total phosphorus concentration at each time step to estimate temporal standard deviation, autocorrelation at-lag-1 and skewness for each level of phosphorus input rate  $\alpha$ . We used a noise intensity  $\sigma$  of 0.25 and we explored two different levels of heterogeneity: (1) no spatial heterogeneity ( $p_{i,j}$  equal in all cells), and (2) with spatial heterogeneity ( $r_{i,j}$  drawn from a uniform distribution [0.8, 1.2]) for three different levels of connectivity (no connectivity *D*=0, weak connectivity *D*=0.1, and strong connectivity *D*=1).

71 Whereas flickering made it difficult to interpret indicators in a monitored time series, flickering 72 in the spatially extended model was more informative. We found that both spatial and temporal 73 indicators perform rather well in identifying a transition. In the simplest case where there was no spatial connectivity (no dispersal, D = 0) in a homogenous environment ( $r_{ij} = r$ ), each spatial unit 74 75 (hereafter called site) flickered individually in time as the phosphorus input rate increased. 76 Monitoring the state of all 2,500 sites in our spatial landscape yielded flickering in space (upper 77 panels in Fig. A5). The transition to eutrophication in this unconnected environment occured 78 gradually (Fig. A5a). Contrary to previous results (Van Nes and Scheffer 2005), this gradual 79 increase in P concentration did not mean that some sites *permanently* shifted to the alternative 80 state, but that they were flickering. That is why the spatial system started to shift way before 81 the actual transition threshold (Fig. A5a). Interestingly, spatial standard deviation increased up to 82 the onset of flickering (Fig. A5b), whereas spatial correlation was 0 (due to the lack of spatial 83 interactions, (Dakos et al. 2010)), and spatial skewness diverged only after flickering. Temporal 84 standard deviation of total spatial P concentration rose, as well as temporal autocorrelation and to 85 a less extend temporal skewness (Fig. A5e-g). All indicators peaked at the point where all sites 86 shifted to the alternative equilibrium and the transition was complete (Fig. A5b-d).

88 Looking at the consequences of flickering in a weakly connected scenario (D = 0.1), we observed 89 similar trends in both spatial and temporal indicators (Fig. A6) with the addition that now also 90 spatial correlation gradually increased and peaked at the transition (Fig. A6c). Interestingly, the 91 increase in connectivity had also two consequences similar to previous findings (Van Nes and 92 Scheffer 2005): the transition to eutrophication became more abrupt, and it occurred closer to the 93 actual input phosphorus threshold (Fig. A6a, compare D = 0 to D = 0.1). This implied that the 94 spatial interactions were actually dominating the individual flickering of the sites, maintaining the 95 system as a whole to the oligotrophic attractor and restricting flickering. Indeed, fewer sites 96 flickered prior to the transition and the ones that did flickered only close to the transition (Online 97 Resource 3, 4). Flickering was suppressed altogether in a strongly connected system (D = 1) (Fig. 98 A6a) (Online Resource 5, 6), while all indicators signaled the transition. Despite the strong noise 99 regime that can flicker individual sites in the landscape when not connected, spatial exchange is 100 homogenizing differences and makes sites behave in unison especially when they are strongly 101 connected (Van Nes and Scheffer 2005; Dakos et al. 2010). Bel et al (2012) have shown in a 102 recent study that similar behavior is observed also in spatial systems exhibiting spatial patterns 103 when disturbed. Under such strong connected conditions flickering vanishes and regardless of the 104 strong noisy regime all indicators signal the approaching transition.

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The progressive suppression of flickering for increasing system connectivity was also identified by potential analysis that marked the existence of the two alternative states (Fig. A7). Although this approach has been manifested in identifying alternative states based on heterogeneous conditions in resilience across the landscape in savanna ecosystems and in boreal forests (Hirota et al. 2011; Scheffer et al. 2012), here we show that not only heterogeneous conditions in

- 111 resilience (Fig. A3), but also changes in underlying conditions under flickering could reveal the
- 112 underlying map of alternative attractors in space.
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- 114 Similar patterns in the temporal and spatial indicators were observed also when we introduced
- 115 heterogeneity in the environment  $(r_{ij} \neq r)$  (Fig. A8).
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### 117 **References**

- Carpenter SR, Brock WA (2010) Early warnings of regime shifts in spatial dynamics using the
   discrete Fourier transform. Ecosphere, 1, art10
- Dakos V, Kéfi S, Rietkerk M, Van Nes EH, Scheffer M (2011) Slowing down in spatially
   patterned ecosystems at the brink of collapse. Am Nat 177: E153–66
- Dakos V, Van Nes E, Donangelo R, Fort H, Scheffer M, Nes EH Van (2010) Spatial correlation
   as leading indicator of catastrophic shifts. Theor Ecol 3: 163–174
- Guttal V, Jayaprakash C (2009) Spatial variance and spatial skewness: leading indicators of
   regime shifts in spatial ecological systems. Theor Ecol 2: 3–12
- Hirota M, Holmgren M, Van Nes EH, Scheffer M (2011) Global resilience of tropical forest and
   savanna to critical transitions. Science 334: 232–235
- Keitt TH, Lewis MA, Holt RD (2001) Allee effects, invasion pinning, and species' borders. Am
   Nat 157: 203–216
- Bel G, Hagberg A, Meron E (2012) Gradual regime shifts in spatially extended ecosystems.
   Theor Ecol 5: 591–604
- 132 Van Nes EH, Scheffer M (2005) Implications of spatial heterogeneity for regime shifts in
   133 ecosystems. Ecology 86: 1797–1807
- Okubo A (1980) Diffusion and ecological problems : mathematical models. Biomathematics.
   Springer Verlag, Berlin
- Scheffer M, Bascompte J, Brock WA, Brovkin V, Carpenter SR, Dakos V et al. (2009) Early warning signals for critical transitions. Nature 461: 53–59

- Scheffer M, Hirota M, Holmgren M, Van Nes EH, Chapin FS (2012) Thresholds for boreal biome transitions. PNAS 109:21384–9 139









